Int J Theor Phys (2008) 47: 722–731
DOI 10.1007/s10773-007-9496-x

Energy Distributions of the Szekeres Universes in Teleparallel Gravity

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Received: 13 April 2007 / Accepted: 16 July 2007 / Published online: 9 August 2007

Abstract In order to evaluate the energy distribution (due to matter and fields including gravitation) associated with a space-time model of Szekeres class I and II metrics, we consider the Einstein, Bergmann–Thomson and Landau–Lifshitz energy definitions in the teleparallel gravity (the tetrad theory of gravitation (TG)). We have found that Einstein and Bergmann–Thomson energy distributions give the same results, Landau–Lifshitz distribution is disagree in TG with these definitions. These results are the same as a previous works of Aygün et al., they investigated the same problem by using Einstein, Bergmann–Thomson, Landau–Lifshitz (LL) and Møller energy-momentum complexes in GR. However, both GR and TG are equivalent theories that is the energy densities are the same using different energy-momentum complexes in both theories. Also, our results are support the Cooperstock's hypothesis.

Keywords Szekeres universe · Energy-momentum distribution · Teleparallel gravity

1 Introduction

The issue of energy localization was first discussed during the early years after the development of general relativity and debate continued for decades. There are different attempts to find a general accepted definition of the energy density for the gravitational field. However, there is still no generally accepted definition known. The foremost endeavor was

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made by Einstein [1] who suggested a definition for energy-momentum distribution. Following this definition, many physicists proposed different energy-momentum complexes: e.g. Tolman [2], Landau and Lifshitz [3], Papapetrou [4], Bergmann and Thomson [5], Weinberg [6], Qadir and Sharif [7] and Møller [8]. Except for the Møller definition, others are restricted to calculate the energy and momentum distributions in quasi-Cartesian coordinates to get a reasonable and meaningful results. The issue of the energy-momentum localization by use of the energy-momentum complexes was revived by Virbhadra's pioneering work [9–24]. Virbhadra and his collaborators verified for asymptotically flat space-times that different energy momentum complexes can give the same result for a given space-time. They also found encouraging results for the case of asymptotically non-flat space-times by using different energy-momentum complexes. Aguirregabiria et al. [25] by using Einstein, Landau–Lifshitz, Papapetrou, Bergmann and Weinberg (ELLPBW) prescriptions, showed that the energy distribution within Kerr–Schild metric is same and Virbhadra [26] found that these five different prescriptions (ELLPBW) prescriptions did not give the same results for the most general non-static spherically symmetric space-time. Also, the problem of energy-momentum localization can also be reformulated in the context of TG. Some authors [27–34] argued that this problem in the context of teleparallel theory of gravity. It has been shown that the results of the two theories agree with each other. By working in the context of TG, Vargas [29] found that the total energy of the closed Friedmann–Robertson–Walker space-time is zero by using TG version of Einstein and Landau–Lifshitz complexes. This agrees with the result obtained by Rosen [38] in GR. Recently Sharif and Amir [39] evaluated the energy-momentum distribution of Lewis–Papapetrou space-times by using the TG version of Møller prescription and found that the results do not agree with those available in the context of GR [40]. Also various authors have investigated in detail energy-momentum problem in the theory of GR and TG by using different space times and different definitions [41–45] and papers cited therein. The basic purpose of this paper is that using the energy-momentum definitions Einstein, Bergmann–Thomson and Landau–Lifshitz in TG to obtain the total energy associated with the Szekeres type I and type II space-times.

The scheme adopted in this paper is follows. In Sect. 2, we briefly present the Szekeres Universes. Then, in Sect. 3, we give Szekeres type II space-time and its tetrad components. In Sect. 4, we give Szekeres type I space-time and its tetrad components. In Sect. 5, we present the energy-momentum definitions of Einstein, Bergmann–Thomson and Landau–Lifshitz in TG. In Sect. 6 we will get the Szekeres class I and class II solutions. Finally, Sect. 7 is devoted to concluding remarks. Throughout this paper we choose units such that $G = 1$ and $c = 1$ and follow the convention that indices take values from 0 to 3 otherwise stated.

2 The Szekeres Class I and Szekeres Class II Space-Times

Szekeres [46] derived a remarkable set of inhomogeneous exact solutions of Einstein's field equations without cosmological constant. The source of curvature of the models is an expanding, irrotational, and geodesic dust. These solutions are divided into two classes usually denoted by I and II. The class I solutions are usually presented in a way that is formally analogous to the Tolman–Bondi spherically-symmetric solutions, which they generalize. This class of solutions has primarily been used to model non-spherical collapse of an inhomogeneous dust cloud [47]. The class II solutions are usually considered as generalizations of the Kantowski–Sachs [48] and Friedmann–Robertson–Walker (FRW) solutions and have primarily been studied as cosmological models [49]. Those of class II are more important as cosmological models, because they can closely approximate, over a finite time interval, the FRW dust models.

In this section, we introduce the Szekeres class II and Szekeres class I metrics and then using these space-times we make some required calculations.

3 The Szekeres Class II Model

The Szekeres class II space-time is defined by the line element [50]

$$ds^2 = -dt^2 + Q^2 dx^2 + R^2(dy^2 + h^2 dz^2) \tag{1}$$

where $Q = Q(x, y, z, t)$, $R = R(t)$ and $h = h(y)$ are functions to be determined. For the line element (1), $g_{\mu\nu}$ is defined by

$$(g_{\mu\nu}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & Q^2 & 0 & 0 \\ 0 & 0 & R^2 & 0 \\ 0 & 0 & 0 & (Rh)^2 \end{pmatrix} \tag{2}$$

and its inverse $g^{\mu\nu}$

$$(g^{\mu\nu}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & Q^{-2} & 0 & 0 \\ 0 & 0 & R^{-2} & 0 \\ 0 & 0 & 0 & (Rh)^{-2} \end{pmatrix}. \tag{3}$$

The non-trivial tetrad field induces a tele-parallel structure on space-time which is directly related to the presence of the gravitational field, and the Riemannian metric arises as

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu. \tag{4}$$

Using this relation, we obtain the tetrad components:

$$h^a{}_\mu = \mathrm{diag}(1, Q, R, Rh) \tag{5}$$

and its inverse is

$$h_a{}^\mu = \mathrm{diag}\left(1, \frac{1}{Q}, \frac{1}{R}, \frac{1}{(Rh)}\right). \tag{6}$$

For the line-element which describes the Szekeres Type II space universe, one can introduce the tetrad basis

$$\theta^0 = dt, \qquad \theta^1 = Qdx, \qquad \theta^2 = Rdy, \qquad \theta^3 = (Rh)dz. \tag{7}$$

4 The Szekeres Class I Model

The Szekeres class I space-time is defined by the line element

$$ds^2 = -dt^2 + e^{2B}(dx^2 + dy^2) + e^{2A} dz^2 \tag{8}$$

where $A = A(x, y, z, t)$, $B = B(x, y, z, t)$ are functions to be determined.

$$(g_{\mu\nu}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & e^{2B} & 0 & 0 \\ 0 & 0 & e^{2B} & 0 \\ 0 & 0 & 0 & e^{2A} \end{pmatrix} \tag{9}$$

and its inverse $g^{\mu\nu}$

$$(g^{\mu\nu}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & e^{-2B} & 0 & 0 \\ 0 & 0 & e^{-2B} & 0 \\ 0 & 0 & 0 & e^{-2A} \end{pmatrix}. \tag{10}$$

The non-trivial tetrad field induces a tele-parallel structure on space-time which is directly related to the presence of the gravitational field, and the Riemannian metric arises as

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu. \tag{11}$$

Using this relation, we obtain the tetrad components:

$$h^a{}_\mu = \mathrm{diag}(1, e^B, e^B, e^A). \tag{12}$$

and its inverse is

$$h_a{}^\mu = \mathrm{diag}\left(1, \frac{1}{e^B}, \frac{1}{e^B}, \frac{1}{e^A}\right). \tag{13}$$

For the line-element which describes the Szekeres Type I space universe, one can introduce the tetrad basis

$$\theta^0 = dt, \qquad \theta^1 = e^B dx, \qquad \theta^2 = e^B dy, \qquad \theta^3 = e^A dz. \tag{14}$$

5 Energy Distributions in Tele-Parallel Gravity

The tele-parallel gravity is an alternative approach to gravitation and corresponds to a gauge theory for the translation group based on Weitzenböck geometry [51]. In the theory of the tele-parallel gravity, gravitation is attributed to torsion [52], which plays the role of a force [53], and the curvature tensor vanishes identically. The essential field is acted by a nontrivial tetrad field, which gives rise to the metric as a by-product. The translational gauge potentials appear as the nontrivial item of the tetrad field, so induces on space-time a tele-parallel structure which is directly related to the presence of the gravitational field. The interesting place of tele-parallel gravity is that, due to its gauge structure, it can reveal a more appropriate approach to consider some specific problem. This is the situation, for example, in the energy and momentum problem, which becomes more transparent when considered from the tele-parallel point of view.

The Einstein, Bergmann–Thomson and Landau–Lifshitz's energy-momentum complexes in tele-parallel gravity [29] are respectively:

$$hE^\mu{}_\nu = \frac{1}{4\pi} \partial_\lambda (U_\nu{}^{\mu\lambda}), \tag{15}$$

$$hB^{\mu\nu} = \frac{1}{4\pi}\partial_\lambda(g^{\mu\beta}U_\beta{}^{\nu\lambda}), \tag{16}$$

$$hL^{\mu\nu} = \frac{1}{4\pi}\partial_\lambda(hg^{\mu\beta}U_\beta{}^{\nu\lambda}) \tag{17}$$

where $U_\beta{}^{\nu\lambda}$ is the Freud's super-potential, which is given by:

$$U_\beta{}^{\nu\lambda} = hS_\beta{}^{\nu\lambda} \tag{18}$$

where $h = \det(h^a{}_\mu)$ and $S^{\mu\nu\lambda}$ is the tensor

$$S^{\mu\nu\lambda} = m_1 T^{\mu\nu\lambda} + \frac{m_2}{2}(T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) + \frac{m_3}{2}(g^{\mu\lambda}T^{\beta\nu}{}_\beta - g^{\nu\mu}T^{\beta\lambda}{}_\beta) \tag{19}$$

with m_1, m_2 and m_3 the three dimensionless coupling constants of tele-parallel gravity [54, 55]. For the tele-parallel equivalent of general relativity the specific choice of these three constants are:

$$m_1 = \frac{1}{4}, \qquad m_2 = \frac{1}{2}, \qquad m_3 = -1. \tag{20}$$

To calculate this tensor, firstly we must calculate Weitzenböck connection:

$$\Gamma^\alpha{}_{\mu\nu} = h_a{}^\alpha \partial_\nu h^a{}_\mu \tag{21}$$

and torsion of the Weitzenböck connection:

$$T^\mu{}_{\nu\lambda} = \Gamma^\mu{}_{\lambda\nu} - \Gamma^\mu{}_{\nu\lambda}. \tag{22}$$

The energy distributions in the complexes of Einstein, Bergmann–Thomson and Landau–Lifshitz in the teleparallel gravity are given by the following equations, respectively,

$$P_\mu^E = \int_\Sigma hE^0{}_\mu dxdydz, \tag{23}$$

$$P_\mu^B = \int_\Sigma hB^0{}_\mu dxdydz, \tag{24}$$

$$P_\mu^L = \int_\Sigma hL^0{}_\mu dxdydz, \tag{25}$$

P_μ is called the momentum four-vector, P_i give momentum components P_1, P_2, P_3 and P_0 gives the energy and the integration hyper-surface Σ is described by $x^0 = t =$ constant.

6 Solutions for Szekeres Type I and II Space-Times

This section gives us the energy and momentum of the universe based on class II and class I metrics in tele-parallel gravity, respectively.

6.1 Solutions in Szekeres Class II Model

Using the above tetrad and its inverse in (5) and (6), we get the following non-vanishing Weitzenböck connection components

$$
\begin{aligned}
&{\Gamma^1}_{11}=\frac{Q_x}{Q}, \qquad {\Gamma^1}_{12}=\frac{Q_y}{Q},\\
&{\Gamma^1}_{13}=\frac{Q_z}{Q}, \qquad {\Gamma^1}_{10}=\frac{Q_t}{Q},\\
&{\Gamma^2}_{20}=\frac{R_t}{R}, \qquad {\Gamma^3}_{32}=\frac{h_y}{h},\\
&{\Gamma^3}_{30}=\frac{R_t}{R}
\end{aligned}
\tag{26}
$$

where x,y,z and t indices describe the derivative with respect to x,y,z and t. The corresponding non-vanishing torsion components are found as

$$
\begin{aligned}
&{T^1}_{12}=-{T^1}_{21}=-\frac{Q_y}{Q},\\
&{T^1}_{13}=-{T^1}_{31}=-\frac{Q_z}{Q},\\
&{T^1}_{10}=-{T^1}_{01}=-\frac{Q_t}{Q},\\
&{T^2}_{20}=-{T^2}_{02}=-\frac{R_t}{R},\\
&{T^3}_{23}=-{T^3}_{32}=\frac{h_y}{h},\\
&{T^3}_{30}=-{T^3}_{03}=-\frac{R_t}{R}.
\end{aligned}
\tag{27}
$$

Taking these results into (19), the non-zero energy components of the tensor $S^{\mu\nu\lambda}$ are found as:

$$
\begin{aligned}
&S^{112}=\frac{1}{2}\frac{h_y}{Q^2R^2h}, \qquad S^{110}=-\frac{R_t}{Q^2R},\\
&S^{223}=\frac{1}{2}\frac{Q_z}{R^4h^2Q}, \qquad S^{220}=-\frac{1}{2}\frac{(RQ)_t}{R^3Q},\\
&S^{323}=-\frac{1}{2}\frac{Q_y}{R^4h^2Q}, \qquad S^{330}=-\frac{1}{2}\frac{(QR)_t}{R^3h^2Q},\\
&S^{020}=\frac{1}{2}\frac{(Qh)_y}{R^2Qh}, \qquad S^{030}=\frac{1}{2}\frac{Q_z}{R^2h^2Q},\\
&S^{002}=-\frac{1}{2}\frac{(Qh)_y}{R^2Qh}, \qquad S^{003}=-\frac{1}{2}\frac{Q_z}{R^2h^2Q}.
\end{aligned}
\tag{28}
$$

Using these components and (15), (16) and (17), we obtain the energy distributions of the Einstein, Bergmann–Thomson and Landau–Lifshitz prescriptions respectively, as follows

$$hE_0^0 = \frac{1}{8\pi}\frac{2hQ_yh_y + h^2Q_{yy} + hQh_{yy} + Q_{zz}}{h}, \tag{29}$$

$$hB_0^0 = \frac{1}{8\pi}\frac{2hQ_yh_y + h^2Q_{yy} + hQh_{yy} + Q_{zz}}{h}, \tag{30}$$

$$hL_0^0 = \frac{1}{8}\frac{R^2(Q_y^2h^2 + 4QQ_yhh_y + h_y^2Q^2 + QQ_{yy}h^2 + Q^2hh_{yy} + Q_z^2 + QQ_{zz})}{\pi}. \tag{31}$$

These results are agree with the previous results obtained by Aygün et al. [56] in general relativity by using these different energy complexes.

6.2 Solutions in Szekeres Class I Model

Using the above tetrad and its inverse in (5) and (6), we get the following non-vanishing Weitzenböck connection components

$$\begin{aligned}
&\Gamma^1{}_{11} = \Gamma^2{}_{21} = B_x, && \Gamma^3{}_{31} = A_x,\\
&\Gamma^1{}_{12} = \Gamma^2{}_{22} = B_y, && \Gamma^3{}_{32} = A_y,\\
&\Gamma^1{}_{13} = \Gamma^2{}_{23} = B_z, && \Gamma^3{}_{33} = A_z,\\
&\Gamma^1{}_{10} = \Gamma^2{}_{20} = B_t, && \Gamma^3{}_{34} = A_t
\end{aligned} \tag{32}$$

where x, y, z and t indices describe the derivative with respect to x, y, z and t. The corresponding non-vanishing torsion components are found

$$\begin{aligned}
&T^1{}_{12} = -T^1{}_{21} = -B_y, \qquad T^2{}_{21} = -T^2{}_{12} = -B_x\\
&T^1{}_{13} = -T^1{}_{31} = T^2{}_{23} = -T^2{}_{32} = -B_z,\\
&T^1{}_{10} = -T^1{}_{01} = T^2{}_{20} = -T^2{}_{02} = -B_t,\\
&T^3{}_{13} = -T^3{}_{31} = -A_x, \qquad T^3{}_{23} = -T^3{}_{32} = A_y,\\
&T^3{}_{30} = -T^3{}_{03} = -A_t.
\end{aligned} \tag{33}$$

Taking these results into (19), the non-zero energy components of the tensor $S^{\mu\nu\lambda}$ are found as:

$$\begin{aligned}
&S^{112} = \frac{1}{2}e^{(-4B)}A_y, \qquad S^{113} = S^{223} = \frac{1}{2}B_ze^{-2(A+B)},\\
&S^{110} = S^{220} = -\frac{1}{2}e^{(-2B)}(B_t + A_t), \qquad S^{212} = -\frac{1}{2}e^{(-4B)}A_x,\\
&S^{313} = -\frac{1}{2}B_xe^{-2(A+B)}, \qquad S^{323} = -\frac{1}{2}B_ye^{-2(A+B)},\\
&S^{330} = -e^{(-2A)}B_t, \qquad S^{010} = \frac{1}{2}(B_x + A_x)e^{-2B},\\
&S^{020} = \frac{1}{2}(B_y + A_y)e^{-2B}, \qquad S^{003} = -\frac{1}{2}\frac{Q_z}{R^2h^2Q}.
\end{aligned} \tag{34}$$

Using these components and (15), (16) and (17) we obtain the energy distributions of the Einstein, Bergmann–Thomson and Landau–Lifshitz respectively, as follows

$$hE_0^0 = \frac{1}{8\pi}[e^A(B_x A_x + A_x^2 + B_{xx} + A_{xx} + A_y B_y + A_y^2 + B_{yy} + A_{yy}) + 2e^{2B-A}(2B_z^2 - B_z A_z + B_{zz})], \tag{35}$$

$$hB_0^0 = \frac{1}{8\pi}[e^A(B_x A_x + A_x^2 + B_{xx} + A_{xx} + A_y B_y + A_y^2 + B_{yy} + A_{yy}) + 2e^{2B-A}(2B_z^2 - B_z A_z + B_{zz})], \tag{36}$$

$$hL_0^0 = \frac{1}{8\pi}[e^{2(B+A)}(B_{xx} + 2B_x^2 + 4B_x A_x + A_{xx} + 2A_x^2 + B_{yy} + 2B_y^2 + 4B_y A_y + A_{yy} + 2A_y^2) + e^{4B}(8B_z^2 + 2B_{zz})], \tag{37}$$

these results are agree with the previous results obtained by Aygün et al. [56] in general relativity.

7 Summary and Discussion

Energy-momentum complexes provide the same acceptable energy-momentum distribution for some systems. However for some systems these prescriptions disagree. The definition of energy-momentum localization in both GR and TG has been very exciting and interesting and a controversial problem. The problem of localization of energy has been re-considered in the frame work of TG by many scientists. The authors [27–39] showed that energy-momentum can also be localized in this theory. It has been shown that the results of two theories can agree with each other. A large number of coordinate dependent as well as coordinate independent definitions of energy and momentum have been given in literature in both TG and GR. Using these different definitions of energy-momentum prescription, several authors studied the energy-momentum distribution for a given space-time. The main object of the present paper is to discuss the energy-momentum localization in TG by using the energy-momentum formulations for inhomogeneous and anisotropic Szekeres class I and II cosmological models and we have considered three different energy complexes in teleparallel gravity: e.g. Bergmann–Thomson, Einstein and Landau–Lifshitz. We found that; (i) the energy distributions in teleparallel gravity for Einstein and Bergmann–Thomson formulations are exactly same in Szekeres class II type space-time and different definitions of these formulations agree with each other. (ii) But we also find that the energy prescription of Landau–Lifshitz disagree in teleparallel gravity with these definitions. (iii) The energy (due to matter plus field) distributions in teleparallel gravity for Einstein and Bergmann–Thomson (BT) formulations are exactly same in Szekeres class I type space-time. (iv) But we also find that the energy prescription of Landau–Lifshitz disagree in Szekeres class I space time with these definitions like Szekeres type II solutions. (v) Also, these results are exactly same as a previous works of Aygün et al. [56]. The authors have investigated the same problem in general relativity by using Einstein, Bergmann–Thomson, Landau–Lifshitz and Møller energy-momentum complexes and found exactly same results in Szekeres class I and II space times. We found that these two gravitational theories give the same results for the total energy (E) distributions in GR and TG for Szekeres type I and II space-times:

$$E_{\mathrm{GR}}^{\mathrm{Eins.}} = E_{\mathrm{TG}}^{\mathrm{Eins.}}, \qquad E_{\mathrm{GR}}^{\mathrm{BT}} = E_{\mathrm{TG}}^{\mathrm{BT}}, \qquad E_{\mathrm{GR}}^{\mathrm{LL}} = E_{\mathrm{TG}}^{\mathrm{LL}}$$

Table 1 The energy densities for Szekeres class I and II space-times in Einstein's theory of GR and TG. Here Θ_0^0, Ξ_0^0, Ω_0^0 represents the Einstein, Bergmann–Thomson and LL energy definitions in GR [56], respectively

Prescriptions	Szekeres type I energy densities in TG and GR
Einstein	$^{TG}hE_0^0 = {}^{GR}\Theta_0^0 = \frac{1}{8\pi}[e^A(B_xA_x + A_x^2 + B_{xx} + A_{xx} + A_yB_y + A_y^2 + B_{yy} + A_{yy}) + 2e^{2B-A}(2B_z^2 - B_zA_z + B_{zz})]$
Bergmann–Thomson	$^{TG}hB_0^0 = {}^{GR}\Xi_0^0 = \frac{1}{8\pi}[e^A(B_xA_x + A_x^2 + B_{xx} + A_{xx} + A_yB_y + A_y^2 + B_{yy} + A_{yy}) + 2e^{2B-A}(2B_z^2 - B_zA_z + B_{zz})]$
Landau–Lifshitz	$^{TG}hL_0^0 = {}^{GR}\Omega_0^0 = \frac{1}{8\pi}[e^{2(B+A)}(B_{xx} + 2B_x^2 + 4B_xA_x + A_{xx} + 2A_x^2 + B_{yy} + 2B_y^2 + 4B_yA_y + A_{yy} + 2A_y^2) + e^{4B}(8B_z^2 + 2B_{zz})]$

Prescriptions	Szekeres type II energy densities in TG and GR
Einstein	$^{TG}hE_0^0 = {}^{GR}\Theta_0^0 = \frac{1}{8\pi}\frac{2hQ_yh_y + h^2Q_{yy} + hQh_{yy} + Q_{zz}}{h}.$
Bergmann–Thomson	$^{TG}hB_0^0 = {}^{GR}\Xi_0^0 = \frac{1}{8\pi}\frac{2hQ_yh_y + h^2Q_{yy} + hQh_{yy} + Q_{zz}}{h}.$
Landau–Lifshitz	$^{TG}hL_0^0 = {}^{GR}\Omega_0^0 = \frac{1}{8}\frac{R^2(Q_y^2h^2 + 4QQ_yhh_y + h_y^2Q^2 + QQ_{yy}h^2 + Q^2hh_{yy} + Q_z^2 + QQ_{zz})}{\pi}$

and we show that these results in Table 1. (vi) From this point of view; both general relativity and teleparallel gravity are equivalent theories, that is the energy densities are the same, using different energy-momentum complexes, in both theories. (vii) From (29–31), (35), (36) and (37) it can be seen that the energy densities are finite and well defined. These results of this paper also support the Cooperstock's hypothesis [57] that energy is localized to the region where the energy-momentum tensor is non-vanishing. We would like to mention here that the results of energy distributions for Szekeres universes are not surprising rather they justify that different energy-momentum complexes, which are pseudo-tensors, are not covariant objects. This is in accordance with the equivalence principle [58] which implies that the gravitational field cannot be detected at a point.

Acknowledgement We would like to thank the referee for their suggestions which helped correcting and improving the final form of the manuscript.

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